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            US Particle Accelerator School
       sponsored by the College of William and Mary
                  Williamsburg, Virginia
        Monday January 19 − Friday January 30, 2004

                LLNL: UCRL−TM−203655
                LBNL: LBNL−54926


List of Files:

  .txt => ascii text format
  .pdf => Adobe Acrobat pdf
  .ppt => Microsoft Power Point (produced on MAC)
  .xls => Microsoft Excel       (produced on MAC)

Class material can be found in the following files and directories:
   JJB − Notes by J.J. Barnard

SML − Notes by S.M. Lund

00_addresses.pdf
  Email and postal addresses of registered students and instructors
  attending the school.

00_addresses_class.pdf
  Email addresses of students in class (this should also be contained in
  00_addresses.pdf)

00_abstract.txt
00_abstract.pdf
00_abstract.ppt
  Class abstract in text, pdf, and power−point formats.

00_cover.pdf
00_cover.ppt
  Cover used in paper printing of class material.

00_outline.txt
00_outline.pdf
  Outline and file list (this file).

00_schedule.pdf
00_schedule.xls
  Actual schedule of class lectures.

00_school_info.pdf
  School information distributed by the USPAS on classes offered,
  registration forms, and local information on Williamsburg, VA.

01_intro.pdf   (JJB)
  Introductory lecture surveying basic concepts.

02_envelope_eqns.pdf  (JJB)
  Introduction to envelope equations.

03_currentlimits.pdf (JJB)
  Introduction to current limits.

04_tran_particle_eqns.pdf  (SML)
  Transverse particle equations of motion.

05_tran_particle_resonances.pdf (SML)
  Transverse particle resonances with application to rings.

06_tran_eq_distributions.pdf  (SML)
  Transverse equilibrium distributions.
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07_injector_longitudinal_I.pdf  (JJB)
  Injectors and longitudinal physics, part I.

08_longitudinal_II.pdf  (JJB)
  Longitudinal physics, part II.

09_longitudinal_III.pdf  (JJB)
  Longitudinal physics, part III.

10_centroid_envelope.pdf  (SML)
  Centroid and envelope evolution including envelope modes and stability.

11_env_modes_halo.pdf  (JJB)
  Continuous focusing envelope modes and beam halo.

12_trans_kinetic_stability.pdf (SML)
  Transverse kinetic stability: conservation constraints, kinetic stability
  bounds, normal modes on a KV beam, and other beam stability topics.

13_pressure_scattering_electrons.pdf  (JJB)
  Vacuum, scattering, and electron effects.

14_heavyionfusion_finalfocus.pdf  (JJB)
  Heavy ion fusion overview and final focus.

15_simulations.pdf  (SML)
  Numerical simulations of beams.

16_JBsummary.pdf  (JJB)
  Summary of lectures by J.J. Barnard.


grades_evaluations  (directory)  Note: This directory is not included
                                       in most distributions.
  grades_evaluations.pdf
    Summary of all grades on problem sets and final and class evaluations.

  Grades.wtest.xls
    Spreadsheet containing problem set grades and distribution info.  Includes
    final grades on JJB problems but not SML problems.

  students.pdf
    Listing of students and institutions, and credit status.

  students_prelim.txt
    Preliminary listing of students signed up before the class along
    with overall statistics of classes from the school.


movies  (directory)

  ESQfastrise_zx.mpg
    3D injector simulation with a fast rise voltage pulse.

  ESQslowrise_zx.mpg
    3D injector simulation with a slow rise voltage pulse.

  hcx.mov
    Simulation of the HCX experiment from the source.

  hollow_movie.mpg
    Simulation on the evolution of a nonuniform density beam.


photos  (directory)

  class_1.jpg      Class photo 1
  class_2.jpg      Class photo 2
  lecture_1.jpg    Lecture photo 1
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  lecture_2.jpg    Lecture photo 2


problems (directory)  Note:  This directory is not included
                             in most distributions.

  01_set1_problems.pdf    Problem  Set #1
  01_set1_solutions.pdf   Solution Set #1

  02_set2_problems.pdf    Problem  Set #2
  02_set2_solutions.pdf   Solution Set #2

  03_set3_problems.pdf    Problem  Set #3
  03_set3_solutions.pdf   Solution Set #3

  04_set4_problems.pdf    Problem  Set #4
  04_set4_solutions.pdf   Solution Set #4

  05_set5_problems.pdf    Problem  Set #5
  05_set5_solutions.pdf   Solution Set #5

  06_set6_problems.pdf    Problem  Set #6
  06_set6_solutions.pdf   Solution Set #6

  07_set7_problems.pdf    Problem  Set #7
  07_set7_solutions.pdf   Solution Set #7

  08_set8_problems.pdf    Problem  Set #8
  08_set8_solutions.pdf   Solution Set #8

  09_final_problems.pdf   Problem  Set #9
  09_final_solutions.pdf  Solution Set #9

  10_replacement_problems.pdf   More difficult replacement problems (not used)


simulations (directory)

  ag-slice.py
     Python input script for example WARP PIC code simulations.   This file
     was used in one interactive class session to carry out example
     simulations by making simple variants of this example run.  See script
     header for instructions on running this script and viewing the output
     files.

  ag-slice.000.cgm
     cgm output file produced by WARP simulation ag-slice.py



Course Outline:

Note:  This outline and the distribution files are arranged in logical
presentation order.  In the actual class this order was deviated from due
to preparation and facility constraints.  The actual order of material
presented can be found in the file:
   00_schedule.pdf


"Intense Beam Physics: Space Charge, Halos, and Related Topics"


John J. Barnard and Steven M. Lund

Lawrence Livermore National Laboratory


1.  Introduction to the Physics of Beams and Basic Parameters  (JJB)
    (01_intro.pdf)
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  1.1 Particle equations of motion
  1.2 Dimensionless parameters: Perveance, phase advance, space charge
      tune depression
  1.3 Plasma physics of beams: collisions, Debye Length
  1.4 Klimontovich equation, Vlasov equation, Liouville's theorem
  1.4 Emittance and brightness


2.  Envelope Equations-I  (JJB)
    (02_envelope_eqns.pdf)

  2.1 Paraxial Ray Equation
  2.2 Envelope equations for axially symmetric beams
  2.3 Cartesian equations of motion
      2.3.1 Quadrupole focusing
      2.3.2 Space charge force for elliptical beams
  2.4 Envelope equations for elliptically symmetric beams


3.  Current Limits in Accelerators and Centroid equations-I (JJB)
    (03_currentlimits.pdf)

  3.1 Axisymmetric beams
     3.1.1 Solenoids
     3.1.2 Einzel Lenses
  3.2 Elliptically symmetric beams
     3.2.1 Derivation of space charge term in envelope equation with
           elliptical symmetry
     3.2.2 Current limit for quadrupoles using Fourier transforms
  3.3 Current limit for continuous focusing
     3.3.1 Calculation of sigma_0 (using matrix multiplication)
     3.3.2 Comparison of quadrupole current limit (from Fourier transform,
     and matrix methods)
  3.4 Centroid equations (first order moments)
     3.4.1 Space charge and focusing forces
  3.5 Image forces (effect on centroid and envelope)


4.  Transverse Particle Equations of Motion (SML)
    (04_tran_particle_eqns.pdf)

  4.1  Particle equations of motion
    4.1.1 Derivation of transverse equations
      - Basic form
      - Including bending and dispersive terms
  4.2  Transverse particle equations of motion in linear focusing channels
    4.1.1 Continuous focusing
    4.2.2 Quadrupole focusing
    4.2.3 Solenoidal focusing
  4.3  Description of applied focusing fields
    4.3.1 Overview
    4.3.2 Multipole descriptions
  4.4  Linear equations of motion without space-charge, acceleration, and
       momentum spread
    4.4.1 Hill's equation
    4.4.2 Orbit stability and eigenvalue structure
  4.5  Floquet's theorem and the phase-amplitude form of the particle orbit
    4.5.1 Floquet's theorem
    4.5.2 Phase amplitude form of the particle orbit
    4.5.3 Particle phase advance
  4.6  The Courant-Snyder invariant and single-particle emittance
    4.6.1 Derivation of the Courant-Snyder invariant
    4.6.2 Interpretation and uses
  4.7 The betatron formulation of the particle orbit
    4.7.1 Formulation
    4.7.2 Envelope of particle orbits
  4.8  Momentum spread effects
    4.8.1 Overview and equations
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    4.8.2 Dispersion function
  4.9  Acceleration and normalized emittance
    4.9.1 Transformation of orbit equations to standard form
    4.9.2 Normalized emittance
  4.10 Example design − The High Current Experiment from LBNL
  References


5.  Transverse Particle Resonances with Application to Rings (SML)
    (05_tran_particle_resonances.pdf)

  5.1 Overview
  5.2 Floquet coordinates
  5.3 Perturbed Hill’s equation in Floquet coordinates
  5.4 Sources and forms of perturbation terms
  5.5 Unperturbed solution and relation to a simple harmonic oscillator
  5.6 Perturbation analysis of perturbed Hill’s equation and resonances
  5.7 Tune restrictions resulting from resonances and machine operating points
  5.8 Effect of space−charge


6.  Transverse Equilibrium Distribution Functions (SML)
    (06_tran_eq_distributions.pdf)

  6.1  Vlasov equation and Vlasov equilibria
    6.1.1 Equilibrium as a function of single−particle constants of the motion
    6.2.2 Typical single−particle constants
  6.2  The KV equilibrium
    6.2.1 Single particle equations of motion with linear space−charge
    6.2.2 Courant−Snyder invariants with linear space−charge
    6.2.3 KV envelope equations
    6.2.4 KV distribution function
    6.2.5 KV depressed phase advance
    6.2.6 Properties of the KV distribution
  6.3  The continuous focusing limit of the KV distribution
    6.3.1 Wavenumbers of particle oscillations
    6.3.2 Distribution function
  6.4  Equilibrium distributions in continuous focusing channels
    6.4.1 Equilibrium form
    6.4.2 Poisson’s equation
    6.4.3 Example distributions
  6.5  The thermal equilibrium distribution in continuous focusing channels
    6.5.1 Overview
    6.5.2 Distribution structure
    6.5.3 Poisson’s equation
    6.5.4 Density profile structure
  6.6  Debye screening in a thermal equilibrium beam
    6.6.1 Poisson’s equation for the perturbed potential due
          to a test particle
    6.6.2 Solution showing characteristic Debye screening
  6.7  The density inversion theorem
    6.7.1 Connection of density to distribution structure in continuous
          focusing channels
    6.7.2 Example application to the KV distribution
  References


7.  Injectors and Longitudinal Physics Part I (JJB)
    (07_injector_longitudinal_I.pdf)

  7.1  Diodes and Injectors
    7.1.1 Space−charge limited flow and child−Langmuir law
    7.1.2 Pierce electrodes
    7.1.3 Transients in injectors and Lampel−Tiefenback solution
  7.2 Injector Choices


8.  Longitudinal Physics Part II (JJB)
    (08_longitudinal_II.pdf)
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  8.1  Acceleration −− introduction
  8.2  Space charge of short bunches (in rf−accelerators)
  8.3  Space charge of long bunches (g−factor model)
  8.4  Longitudinal 1D Vlasov equation
  8.5  Longitudinal fluid equation
  8.4  Longitudinal space charge waves
  8.5  Longitudinal rarefaction waves and bunch end control


9.  Longitudinal Physics Part III (JJB)
    (09_longitudinal_III.pdf  (JJB)

  9.1  Longitudinal cooling from acceleration
  9.2  Longitudinal resistive instability
  9.3  Bunch compression
  9.4  Longitudinal envelope equation
  9.4  Neuffer distribution function


10. Centroid and Envelope Descriptions of Beam Evolution II (SML)
    (10_centroid_envelope.pdf)

  10.1 Overview
  10.2 Derivation of transverse centroid and envelope equations of motion
       for an unbunched beam
    10.2.1 Statistical average
    10.2.2 Particle equations of motion
    10.2.3 Distribution assumptions
    10.2.4 Direct and image self−fields
    10.2.5 Centroid equations
    10.2.6 Envelope equations
  10.3 Centroid equations
    10.3.1 Solution structure
    10.3.2 Image scaling
  10.4 Envelope equations
    10.4.1 Properties of terms
    10.4.2 Matched solution
    10.4.3 Mismatch and mismatch modes
      − Continuous focusing
      − Periodic solenoidal focusing
      − Periodic quadrupole focusing
  10.5 Transport limit scaling based on the matched beam envelope equation for
       periodic focusing channels
    10.5.1 Overview
    10.5.2 Example calculation for a periodic FODO quadrupole
           transport channel
    10.5.3 Discussion on application of formulas in design
    10.5.4 Results of more detailed models
  10.6 Formulation and use of 1st order coupled moment approaches
       (not included)
    10.6.1 Motivation
    10.6.2 Example illustration − dispersive effects
 References


11. Continuous Focusing Envelope Modes and Beam Halo  (JJB)
    (11_env_modes_halo.pdf)

  11.1 Envelope modes of unbunched beams in continuous focusing
  11.2 Envelope modes of bunched beams in continuous focusing
  11.3 Halos from mismatched beams
    11.3.1 What is halo? Why do we care
    11.3.2 Qualitative picture of halo formation: mismatches
           resonantly drive particles to large amplitude
    11.3.3 Core/particle models
    11.3.4 Amplitude phase analysis
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12. Transverse Kinetic Stability (SML)
    (12_trans_kinetic_stability.pdf)

  12.1 Overview
    12.1.1 Collective modes and the envelope model
    12.1.2 Possibilities for collective oscillations beyond the envelope
           model -- qualitative motivation
    12.1.3 Vlasov, continuous focusing model
    12.1.4 Limits of continuous focusing model
  12.2 Linearized Vlasov equation
    12.1 Derivation
    12.2 Formal solution by the method of characteristics
  12.3 Collective modes on a KV equilibrium beam
    12.3.1 Equilibrium characteristics , normal mode perturbations, and
           linear eigenvalue equation for delta_phi
    12.3.2 Gluckstern mode solution for delta_phi: mode dispersion relation
    12.3.3 Properties of eigenfunctions
    12.3.4 Dispersion relation and KV instabilities
  12.4 Global conservation constraints
    12.4.1 Conservation laws and interpretation
    12.4.2 Uses of conservation constraints
  12.5 Kinetic stability theorem
    12.5.1 Conserved free energy
    12.5.2 Expansion in perturbations
    12.5.3 Perturbation bound and a sufficient condition for stability
    12.5.4 Interpretation and example applications
  12.6 Energy extreme states
       (not included in distribution)
    12.6.1 Uniform density beam and energy extrema
    12.6.2 Comments on applicability to elliptical symmetry beams
  12.7 Wangler's theorem
       (not included in distribution)
    12.7.1 Connection of density fluctuations about an rms equivalent
           beam to emittance evolution
    12.7.2 Consequences of theorem for existence of Vlasov equilibria
           with nonuniform density in periodic focusing channels
  12.8 Collective relaxation and rms emittance growth
       (not included in distribution)
    12.8.1 Use of conservation constraints to bound emittance growth
           in relaxation from an arbitrary initial beam to a final
           uniform density mismatched beam
      - Space charge components
      - Mismatch components
    12.8.1 Example applications
    12.8.2 Bound on further changes under relaxation to full thermal
            equilibrium
  12.9 Landau damping of transverse kinetic modes
       (not included in distribution)
  References


13. Pressure, Scattering, and Electron Effects (JJB)
    (13_pressure_scattering_electrons.pdf)

  13.1 Beam/beam Coulomb collisions
  13.2 Beam/residual-gas scattering
  13.3 Charge-changing processes
  13.4 Wall effects
    13.4.1 gas pressure instability
  13.5 Electron cloud processes
    13.5.1 Multiple-bunch beam-induced multipacting
    13.5.2 Single-bunch beam-induced multipacting
  13.6 Electron-ion instability


14. Heavy Ion Fusion and Final Focus (JJB)
    (14_heavyionfusion_finalfocus.pdf)

  14.1 An application of intense beams: Heavy Ion Fusion
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    14.1.1 Requirements
    14.1.2 Targets for inertial confinement fusion
    14.1.3 Accelerator
    14.1.4 Drift compression
    14.1.5 Final focus
  14.2 Final focus
    14.2.1 Predicting spot size using envelope equation
           and estimate of effects from chromaticity
  14.3 Experiments for Heavy Ion Fusion


15. Numerical Simulations (SML)
   (15_simulations.pdf)

  15.1 Why numerical simulation?
  15.2 Classes of intense beam simulations
    15.2.1 Particle methods
    15.2.2 Distribution methods
    15.2.3 Moment methods
  15.3 Numerical methods
    15.3.1 Discretizations
      - Derivatives
      - Integrals
    15.3.1 Application to moment methods
      - Euler and Runge-Kutta advances
      - Example application
  15.4 Numerical methods for particle and distribution methods
    15.4.1 Field discretizations
    15.4.2 Particle methods and PIC codes for solving Valsov's equation
      - Leapfrog advance
      - Field solution
      - Particle weighting
      - Advance cycle
      - Initialization
      - Numerical convergences
      - Examples
    15.4.1 Distribution methods
      - Similarities with PIC method
      - Distribution advance
      - Examples
  15.5 Overview of the WARP PIC code
  References


16. Summary of Lectures by John J. Barnard (JJB)
    (16_JBsummary.pdf)

  16.1 Emittance and phase space review
  16.2 Particle equations of motion (radial and Cartesian)
  16.3 Summary of 6 statistical envelope equations and two equations based
       on particular distribution functions
  16.4 Current limits
  16.5 Using envelope equations to estimate spot size
  16.6 Longitudinal dynamics summary
  16.7 Instability summary
  16.8 Halo summary
  16.9 Electron, gas, pressure, and scattering effects summary
  16.10 Summary of HIF
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